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Doctoral School FNRS

Nonlinear phenomena, complex systems and statistical mechanics

1-2-3 February 2022 - 9h30-12h30 – Université de Namur

Introduction to dynamical systems on complex networks

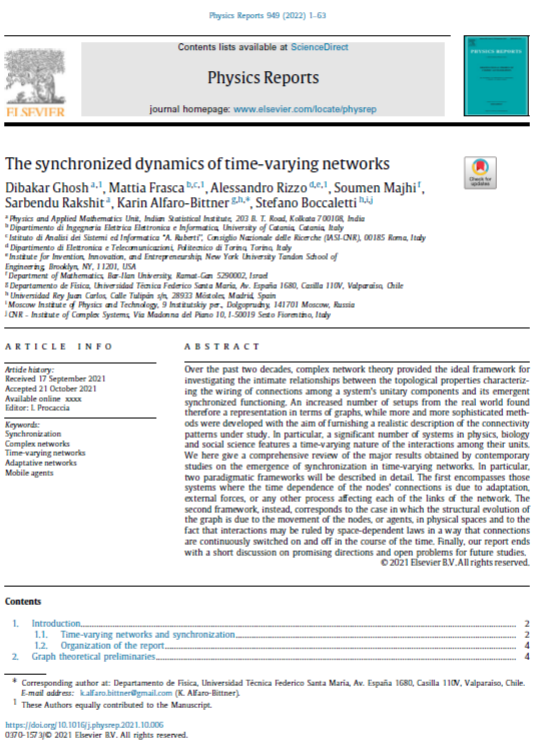
Mattia Frasca (Università di Catania) & Timoteo Carletti (Université de Namur)

Lecture 5 – Synchronization in temporal networks

Mattia Frasca (Univ. of Catania)

Contents

- Models of temporal networks
- Case studies of synchronization in temporal networks
  - – blinking networks
  - – activity driven networks
  - – networks of mobile agents
  - – adaptive networks



Physics Reports 949 (2022) 1–63

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

The synchronized dynamics of time-varying networks

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ARTICLE INFO

Article history:
Received 17 September 2021
Accepted 21 October 2021
Available online xxxx
Editor: I. Procaccia

Keywords:
Synchronization
Complex networks
Time-varying networks
Adaptative networks
Mobile agents

ABSTRACT

Over the past two decades, complex network theory provided the ideal framework for investigating the intimate relationships between the topological properties characterizing the wiring of connections among a system's unitary components and its emergent synchronized functioning. An increased number of setups from the real world found therefore a representation in terms of graphs, while more and more sophisticated methods were developed with the aim of furnishing a realistic description of the connectivity patterns under study. In particular, a significant number of systems in physics, biology and social science features a time-varying nature of the interactions among their units. We here give a comprehensive review of the major results obtained by contemporary studies on the emergence of synchronization in time-varying networks. In particular, two paradigmatic frameworks will be described in detail. The first encompasses those systems where the time dependence of the nodes' connections is due to adaptation, external forces, or any other process affecting each of the links of the network. The second framework, instead, corresponds to the case in which the structural evolution of the graph is due to the movement of the nodes, or agents, in physical spaces and to the fact that interactions may be ruled by space-dependent laws in a way that connections are continuously switched on and off in the course of the time. Finally, our report ends with a short discussion on promising directions and open problems for future studies.

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Contents

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1 These Authors equally contributed to the Manuscript.

https://doi.org/10.1016/j.physrep.2021.10.006

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Contents

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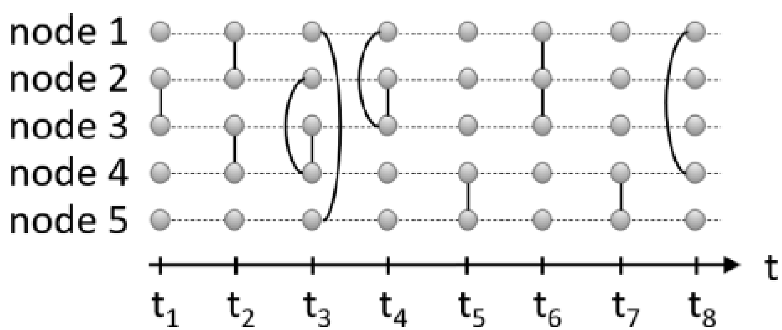
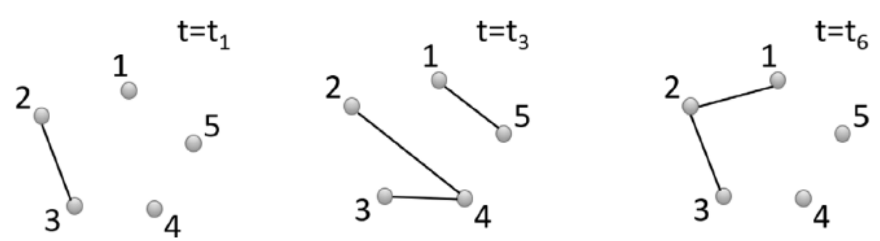
Temporal networks

- V=V(t) and/or E=E(t), with $t=0,\ldots,t_{max}$ (window of observation)
- Without lack of generality, V may be considered time-independent
- Two representations:

Event-based



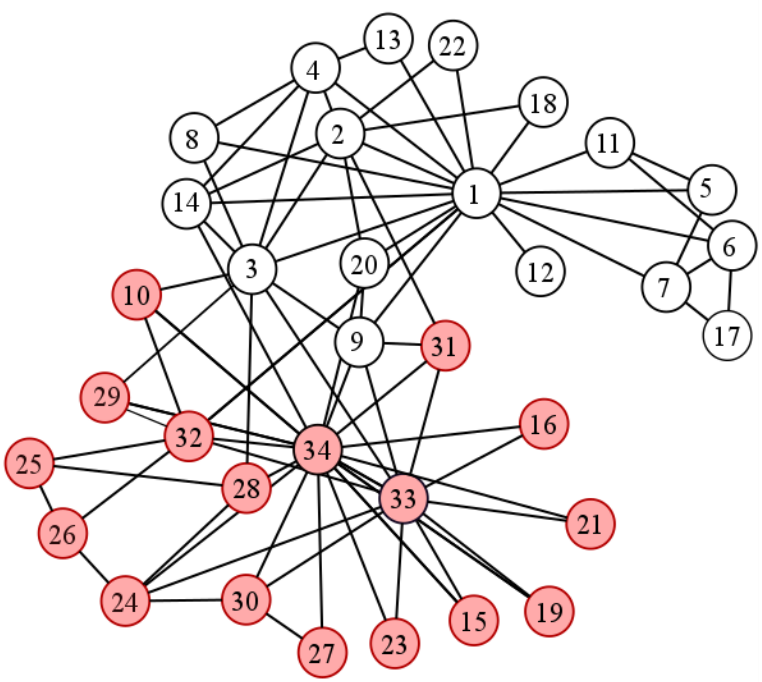
Snapshot

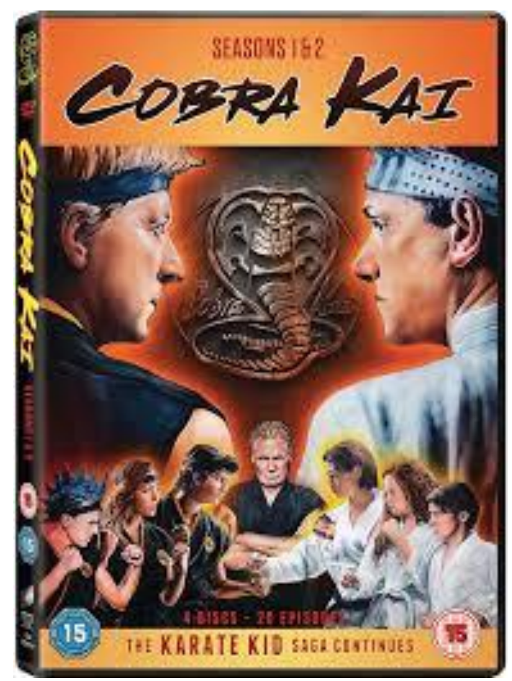


$$A = A(t)$$

Do networks change in time?

- Zachary karate club: well-known example of a complex network, benchmark for community detection, … Is it a static or time-varying network?





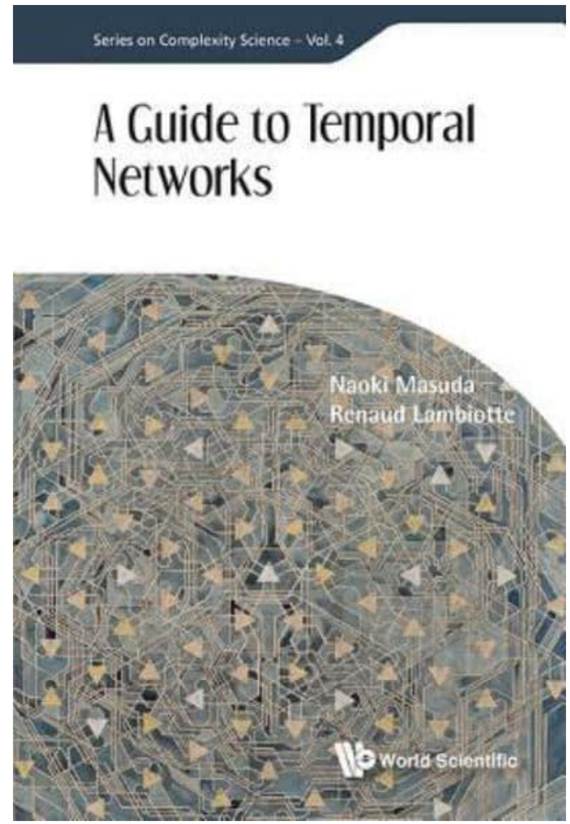
Do networks change in time?

- Zachary karate club: well-known example of a complex network, benchmark for community detection, … Is it a static or time-varying network?
- And the Barabasi-Albert model?
- Networks are subject to growth, aging, fluctuations, …
- It is a matter of which aspects we need to include in the model
- So, ultimately it is a matter of *time scales*
- There are no good or bad models. Models are always relative to their goal

Disclaimer

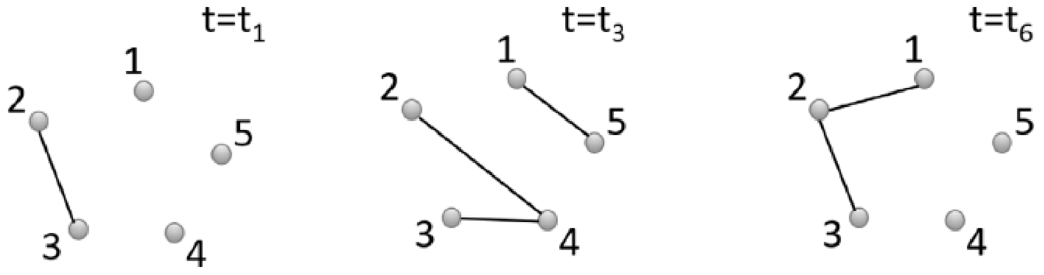
- I will focus here on synthetic models used in studies on synchronization

- Models (often data-driven) aimed at the characterization of the process of link generation at nodes are extensively reviewed in Masuda, Lambiotte, *Guide To Temporal Networks*, World Scientific.



Modeling temporal networks

- Snapshot representation



- Adjacency matrix

$$A = A(t)$$

$A_{ij}(t) = 1$ if at time t nodes i and j are connected by a link
$A_{ij}(t) = 0$ othewise

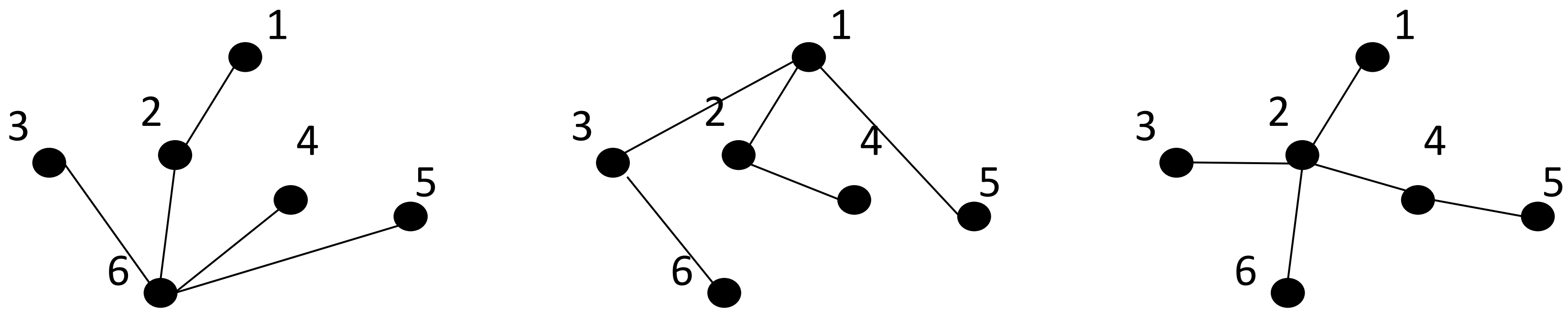
- How A depends on time?

Blinking networks

$$A = A(s(t))$$

where s(t) is a *switching sequence*

- The switching sequence determines which links at each time instant are switched on
- Example 1. At each time instant, the topology is given by an ER network

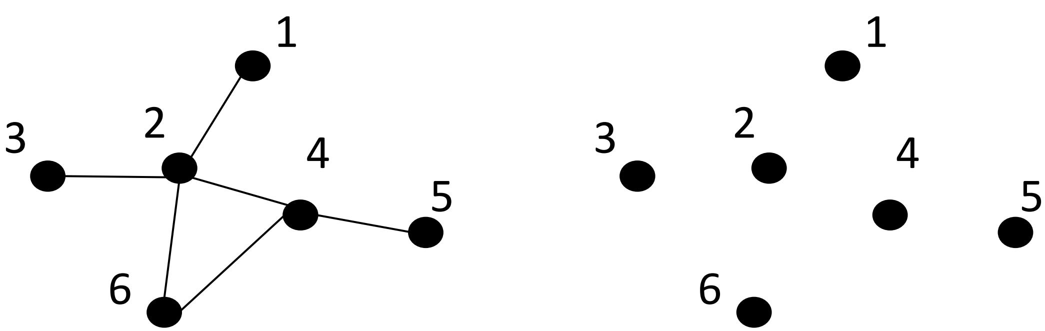


Blinking networks

$$A = A(s(t))$$

where s(t) is a *switching sequence*

- The switching sequence determines which links at each time instant are switched on
- Example 2. On-off coupling



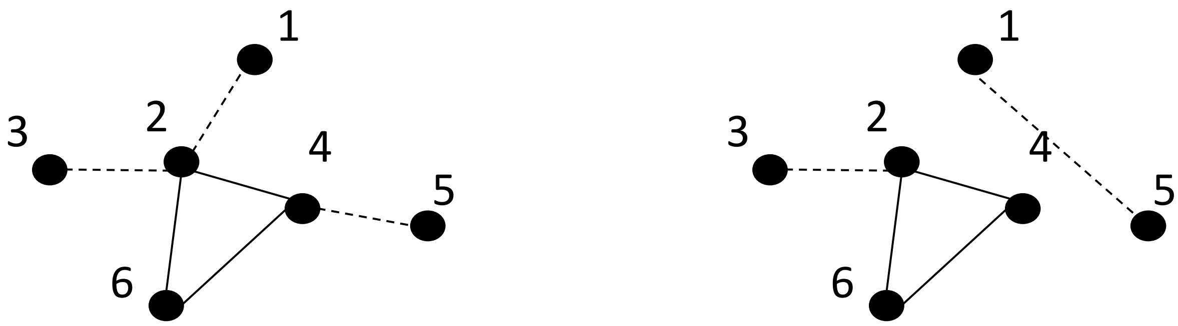
$$A\big(s(t)\big) = \varepsilon(t)A$$

Blinking networks

$$A = A(s(t))$$

where s(t) is a *switching sequence*

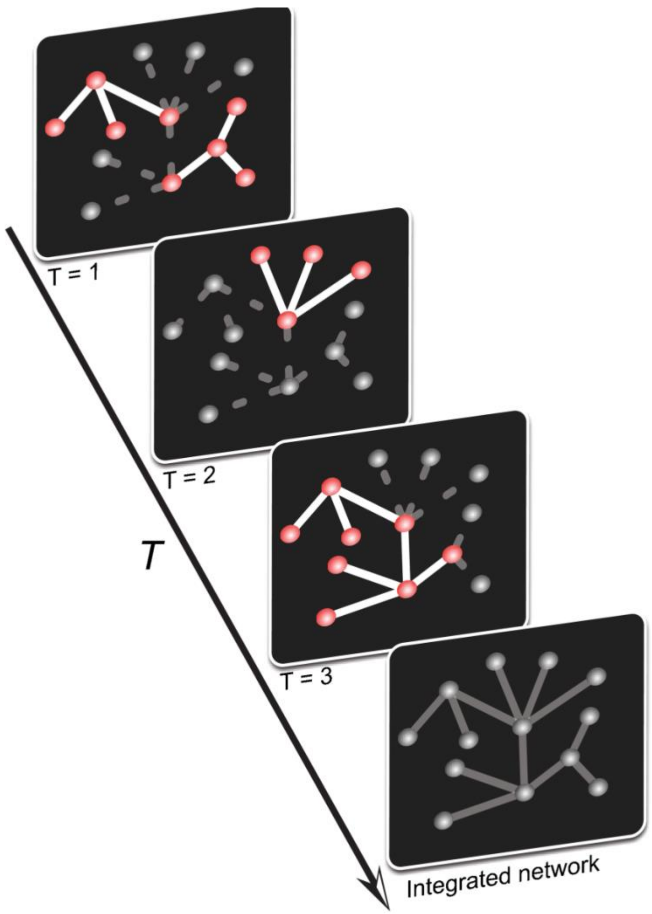
- The switching sequence determines which links at each time instant are switched on
- Example 3. Some connections are switched, others constitute the network fixed backbone



Activity-driven networks

$$A = A(a(t))$$

where a(t) models node activities



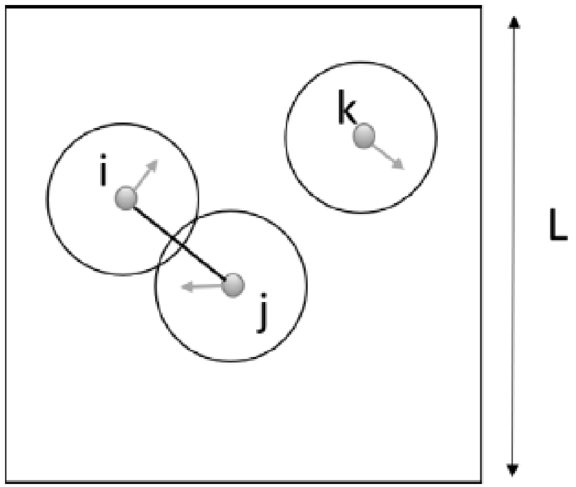
- Agents/nodes have an activity/firing rate ai drawn from a probability distribution F(a)
  - At each time step t, N disconnected nodes are considered
  - With probability a_i each node becomes active and links with m other randomly selected individuals
  - At the next time t+dτ all the links are deleted and a new step of the algorithm for link generation is iterated
- The model fits data from several social networks (scientific collaborations, IMDB, tweeter, …)

Perra, N., Gonçalves, B., Pastor-Satorras, R., & Vespignani, A. (2012). Activity driven modeling of time varying networks. *Scientific Reports,* 2 : 469 | DOI: 10.1038/srep00469.

Networks of mobile agents

$$A = A(y(t))$$

where y(t) are agent positions

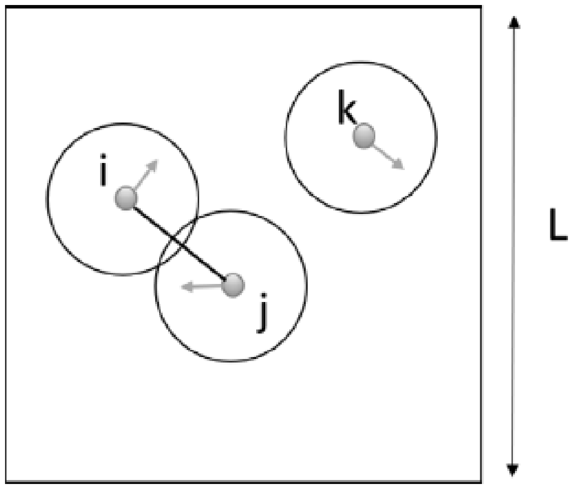


$$\mathcal{A}_{ij}(t) = 1 \Leftrightarrow \|\mathbf{y}_i(t) - \mathbf{y}_j(t)\| \le R.$$

$$\mathcal{A}_{ij}(t) = 1 \Leftrightarrow \sqrt{\left(y_{i,1}(t) - y_{j,1}(t)\right)^2 + \left(y_{i,2}(t) - y_{j,2}(t)\right)^2} \le R.$$

- R is the sensing radius
- The model generalizes the R-disk spatial graph (also known as random geometric graph)
- Beyond the connectivity criterion, the rule of motion needs also to be specified

Rule of motion



- Agent velocity: fixed modulus, variable heading

$$\mathbf{v}_i(t) = v e^{j\theta_i(t)}.$$

- With probability p_j, agents jump into random positions

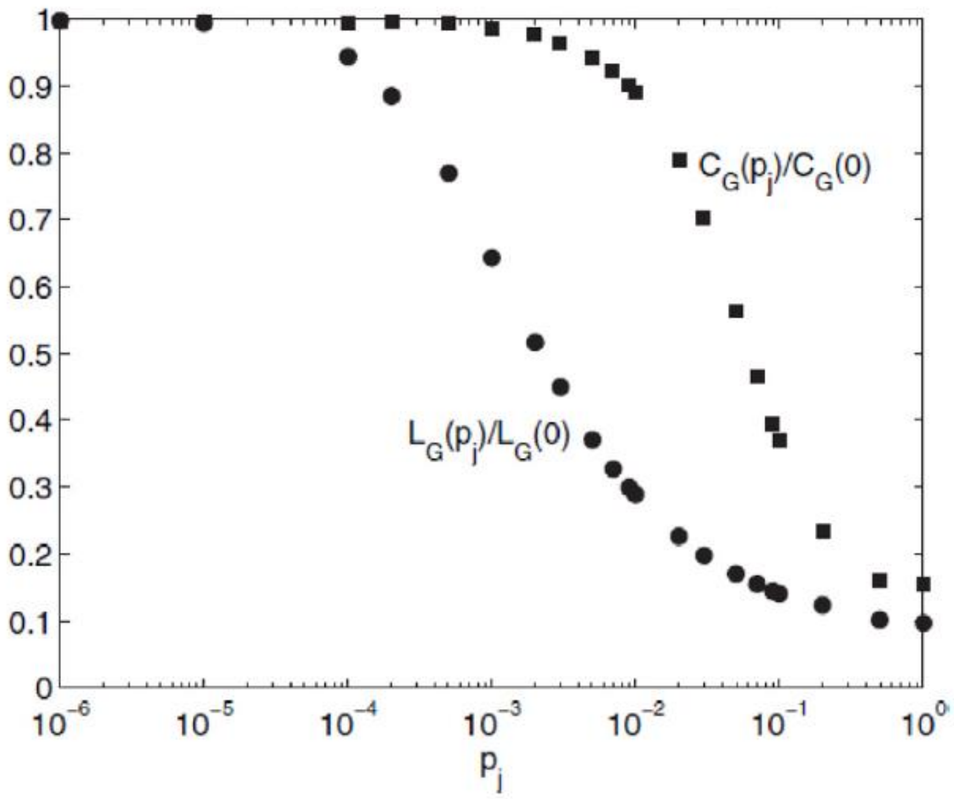
$$\mathbf{y}_i(t_k + \tau_M) = \xi_i(t_k),$$

- With probability $1-p_j$, agents move as random walkers

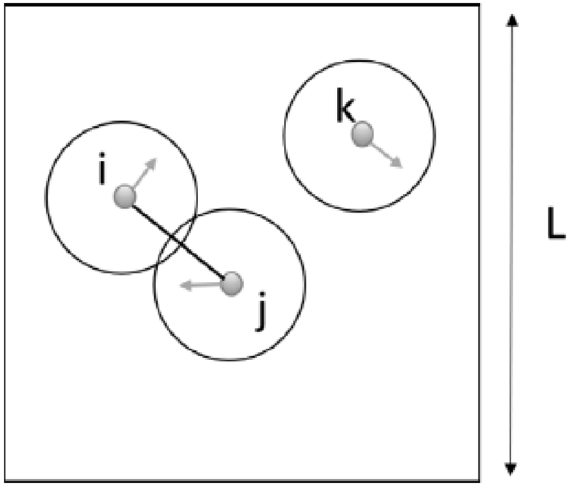
$$\mathbf{y}_i(t_k + \tau_M) = \mathbf{y}_i(t_k) + \tau_M \mathbf{v}_i(t_k),$$
$$\theta_i(t_k) = \eta_i(t_k),$$

An interesting property of the model

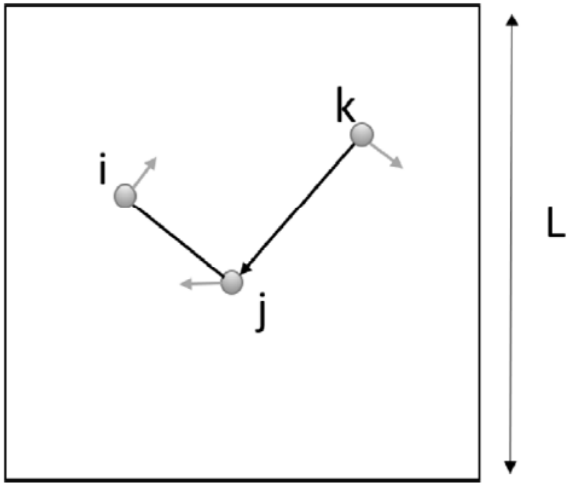
Small-world effect vs. probability of jumps



Metric vs. topological connectivity criterion



i and j are connected if at a distance smaller than R



i is connected with its first m nearest neighbors

Adaptive networks

- Structure and dynamical states coevolve
- Dynamics of the weights:

$$\dot{A}_{ij}(t) = f(x_i(t), x_j(t))$$

- Reshaping the interactions as a function of the environment

Contents

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Stability of the synchronization manifold in temporal networks

- Dynamics

$$\dot{\mathbf{x}}_i(t) = f(\mathbf{x}_i) - \epsilon \sum_{i=1}^{N} \mathcal{L}_{ij}(t) h(\mathbf{x}_j),$$

- Linearization around the synchronization manifold

$$\delta\dot{\mathbf{x}}(t) = [I_N \otimes Jf(\mathbf{x}_s) - \mathcal{L}(t) \otimes Jh(\mathbf{x}_s)]\delta\mathbf{x}(t),$$

- Modes are not decoupled

$$\frac{d\eta_i(t)}{dt} = [Jf(\mathbf{x}_s) - \epsilon\lambda_i(t) Jh(\mathbf{x}_s)]\eta_i(t) - \sum_{j=1}^{N} v_i(t)^T \frac{dv_j(t)}{dt}\eta_j(t),$$

Becomes zero, if

- The network is static
- $L(t_1)$ commutes with $L(t_2)$ and so on…

A general criterion: dynamics under fast-switching

- *Fast Switching Hypotesis (FSH):*

If

i. $\bar{L} = \frac{1}{T}\int_{t}^{t+T} L(\tau)d\tau$ admits synchronization

ii. switching between all the possible networks is sufficiently fast

then the network L(t) will synchronize.

- Hypothesis i. can be checked using the MSF approach

Stilwell, D. J., Bollt, E. M., & Roberson, D. G. (2006). Sufficient conditions for fast switching synchronization in time-varying network topologies. *SIAM Journal on Applied Dynamical Systems,* 5(1), 140-156.

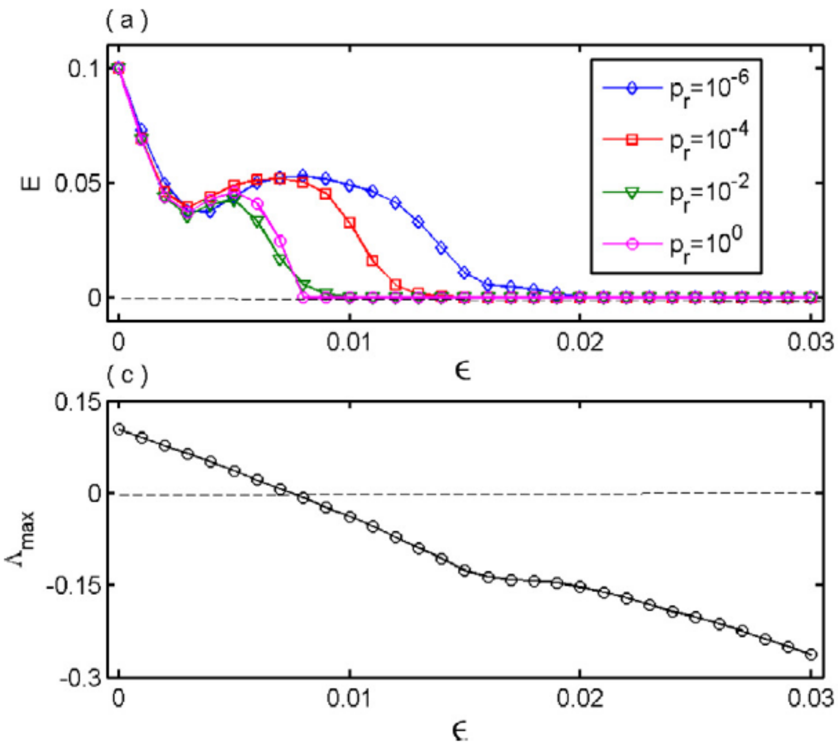
Contents

- Models of temporal networks
- Case studies of synchronization in temporal networks
  - blinking networks
  - activity driven networks
  - networks of mobile agents
  - adaptive networks

Blinking networks

- Network of Rössler oscillators

$$\dot{x}_i(t) = -y_i(t) - z_i(t) - \epsilon \sum_{j=1}^{N} \mathcal{L}_{ij}(t) x_j(t),$$
$$\dot{y}_i(t) = x_i(t) + a y_i(t),$$
$$\dot{z}_i(t) = b + z_i(t)(x_i(t) - c),$$



- Higher rewiring probability enhances synchrony
- Prediction possible in the fast switching regime

Contents

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  - blinking networks
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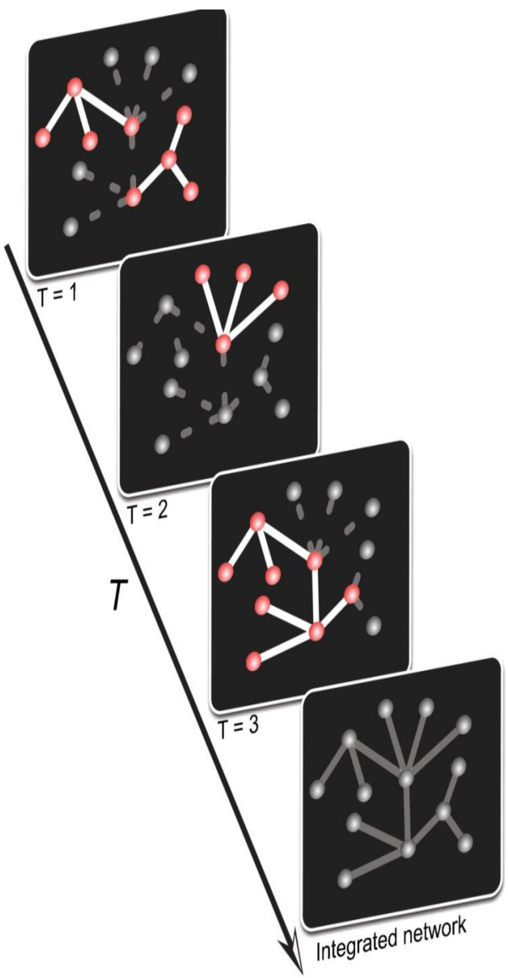
Activity-driven networks: prediction under fast-switching



$$\bar{L} = \frac{1}{T}\int_{t}^{t+T} L(\tau)d\tau \quad \Rightarrow \quad \bar{L} = \frac{1}{M}\sum_{k=1}^{M} L(t_k)$$

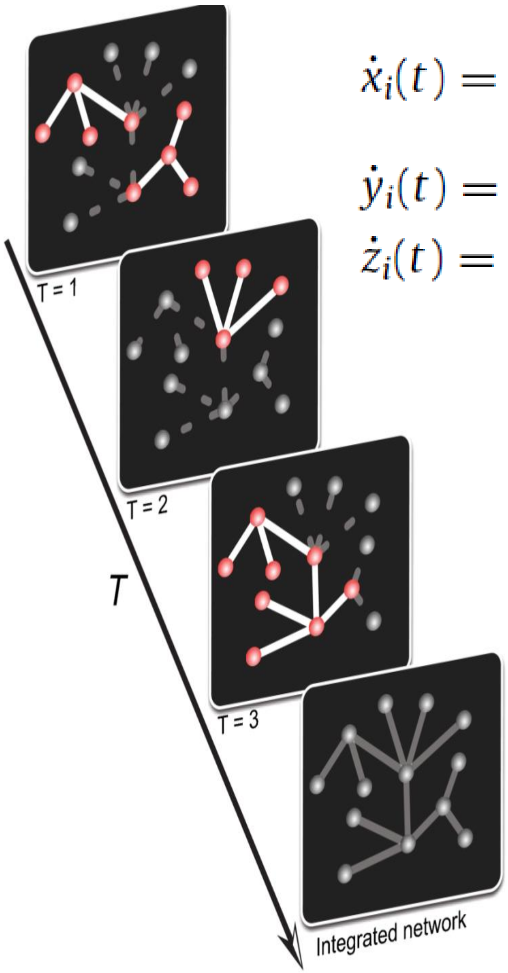
$\Rightarrow$ for large T $\quad \bar{L}_{ij} = p_{ij}$

p_{ij} - probability that i and j are connected at a given time

For activity driven networks:

$$p_{ij} = 1 - \left[\left(1 - \frac{a_i}{N^2}\right)\left(1 - \frac{a_j}{N^2}\right)\right]^{mN}$$

Activity-driven networks

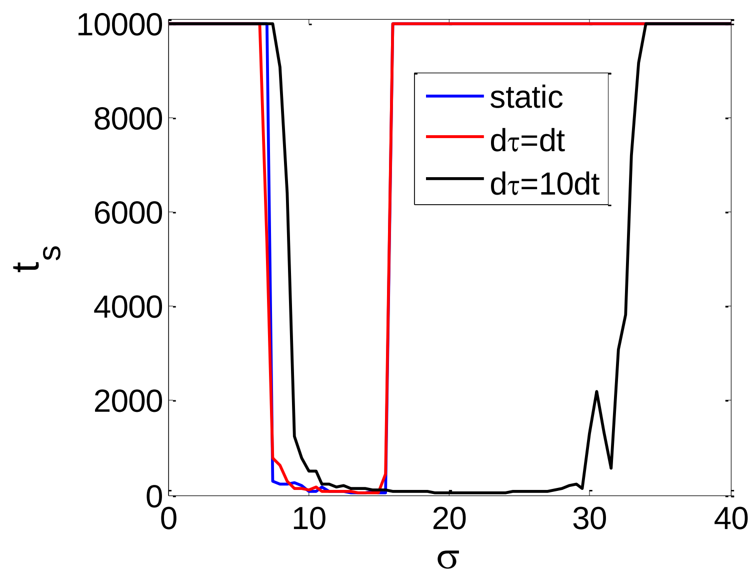


$$\dot{x}_i(t) = -y_i(t) - z_i(t) - \epsilon \sum_{j=1}^{N} \mathcal{L}_{ij}(t) x_j(t),$$
$$\dot{y}_i(t) = x_i(t) + a y_i(t),$$
$$\dot{z}_i(t) = b + z_i(t)(x_i(t) - c),$$

ADN with N=200, m=5, F(a) power law with γ=-2.1

Measure:

- Time to synchronization



Windows of opportunity

- For intermediate switching frequencies there are regions where synchronization is stable in the temporal network, but unstable in the time-average structure (and so in the fast switching regime)

- Other temporal networks show the same behavior

- Interplay between two time-scales, that for local synchronization and that for network reconfiguration

Contents

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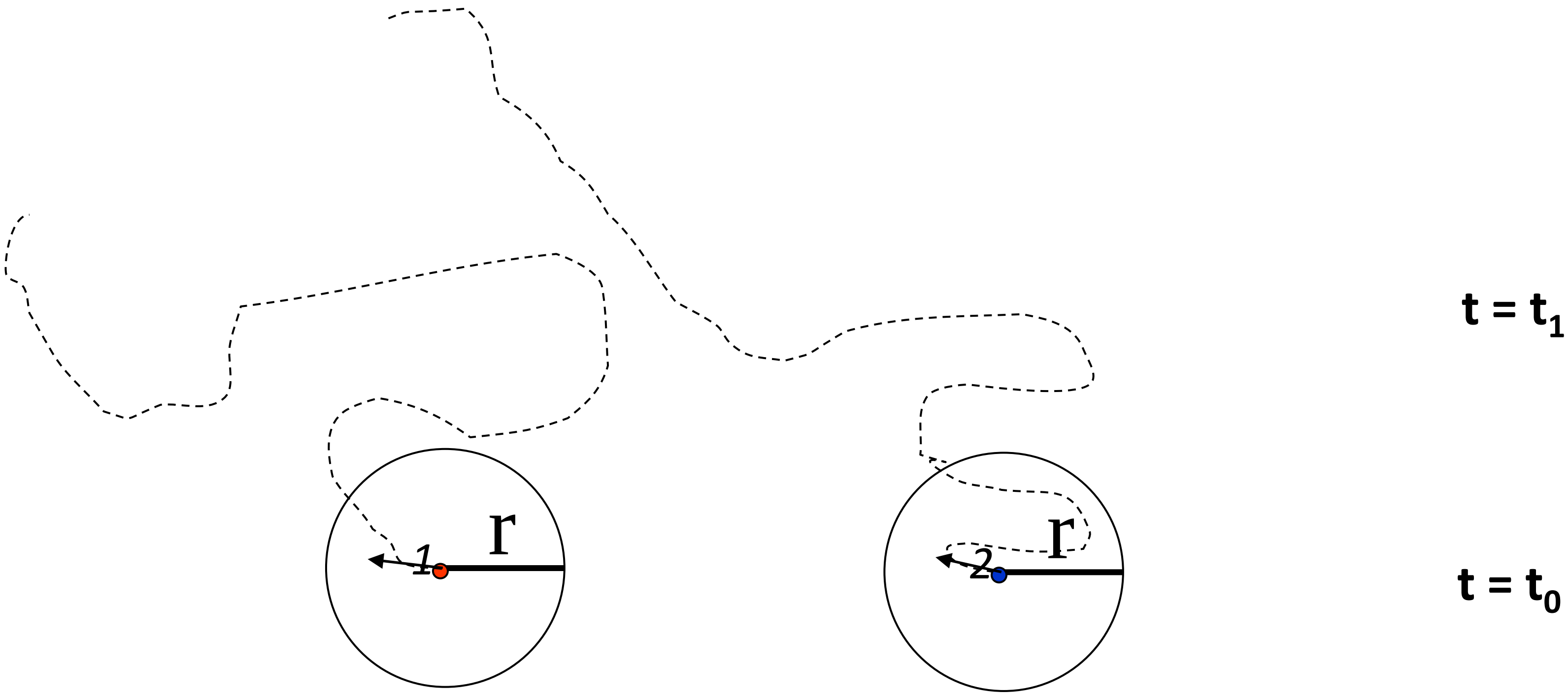
  - blinking networks
  - activity driven networks
  - networks of mobile agents
  - adaptive networks

Networks of mobile oscillators

- Consider the case of N=2 moving chaotic agents

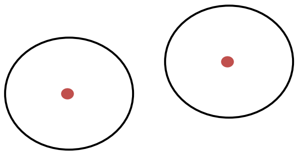
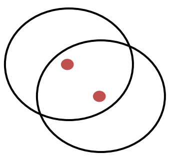


$$\dot{\mathbf{y}}_1 = \mathbf{f}(\mathbf{y}_1) + K(\mathbf{E}\mathbf{y}_2 - \mathbf{E}\mathbf{y}_1)$$

$$\dot{\mathbf{y}}_2 = \mathbf{f}(\mathbf{y}_2) + K(\mathbf{E}\mathbf{y}_1 - \mathbf{E}\mathbf{y}_2)$$

The interaction network is a time-varying network

Network configurations for N=2

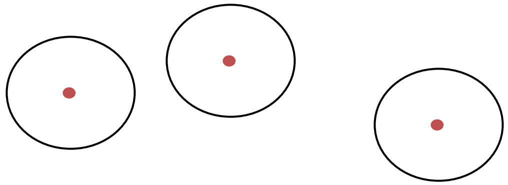
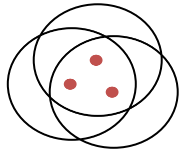
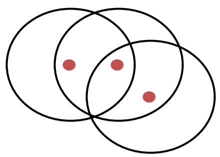
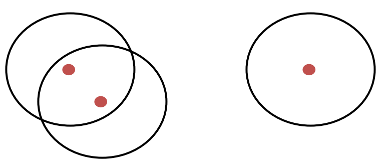
	$G_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	**Not neighbor with probability p_0**
	$G_A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	**Neighbor with probability $p_A = \pi r^2 / L^2$**

$$\overline{G} = p_0 G_0 + p_A G_A = p_A G_A$$

- p_A acts as a coupling parameter →

the non-zero eigenvalue of $\overline{G}$ is $\lambda = 2 p_A$

Network configurations for N=3

	$G_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$G_A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$
	$G_1 = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ $G_2 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ $G_3 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}$
	$G_{12} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $G_{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ $G_{13} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$\boxed{G_1 + G_2 + G_3 = 2G_A}$$

$$\boxed{p_1 = p_2 = p_3}$$

$$\boxed{G_{12} + G_{23} + G_{13} = G_A}$$

$$\boxed{p_{12} = p_{23} = p_{13}}$$

Network configurations for N=3

$$\boxed{G_1 + G_2 + G_3 = 2G_A}$$

$$\boxed{G_{12} + G_{23} + G_{13} = G_A}$$

$$\boxed{p_1 = p_2 = p_3}$$

$$\boxed{p_{12} = p_{23} = p_{13}}$$

$$\boxed{\overline{G} =} \, p_A G_A + \cancel{p_0 G_0} + p_{12} G_{12} + p_{13} G_{13} + p_{23} G_{23} +$$
$$+ p_1 G_1 + p_2 G_2 + p_3 G_3 =$$
$$= p_A G_A + p_1 (G_1 + G_2 + G_3) + p_{12} (G_{12} + G_{13} + G_{23}) =$$
$$= (p_A + 2p_1 + p_{12}) G_A \, \boxed{= pG_A}$$

- p acts as a coupling parameter →

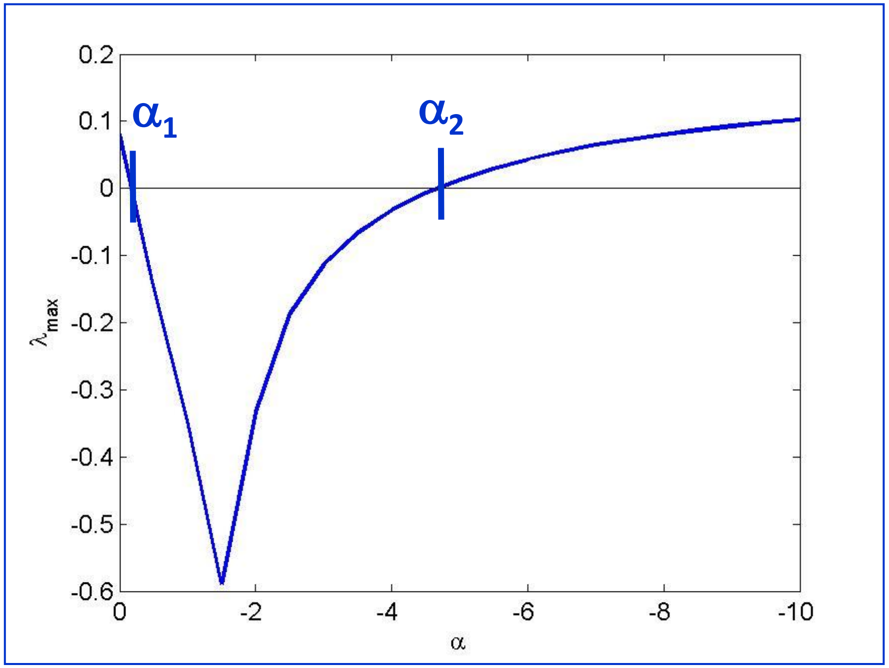
the non-zero eigenvalue of $\overline{G}$ is $\lambda = 3p$

Synchronization conditions under FSH

For N agents → $\overline{G} = pG_A \qquad \overline{\lambda} = pN = \frac{\pi r^2}{L^2} N = \pi r^2 \rho$

- $KN \in [\alpha_1, \alpha_2]$

➡ $\rho > \rho_c = \frac{\alpha_1}{\pi r^2 K}$



- $KN > \alpha_2$

➡ $\frac{\alpha_1}{\pi r^2 K} = \rho_{c_1} < \rho < \rho_{c_2} = \frac{\alpha_2}{\pi r^2 K}$

M. Frasca, A. Buscarino, A. Rizzo, L. Fortuna, S. Boccaletti, "Synchronization of moving chaotic agents", Physical Review Letters, 100, 044102-1-4, 2008.

Synchronization conditions under FSH

Rossler MSF

$$\alpha_1 \cong 0.22 \quad \alpha_2 \cong 4.08$$

$$\delta(t) = \frac{1}{3(N-1)} \sum_{i=2}^{N} \left(\left| x_1^i - x_1^1 \right| + \left| x_2^i - x_2^1 \right| + \left| x_3^i - x_3^1 \right| \right)$$

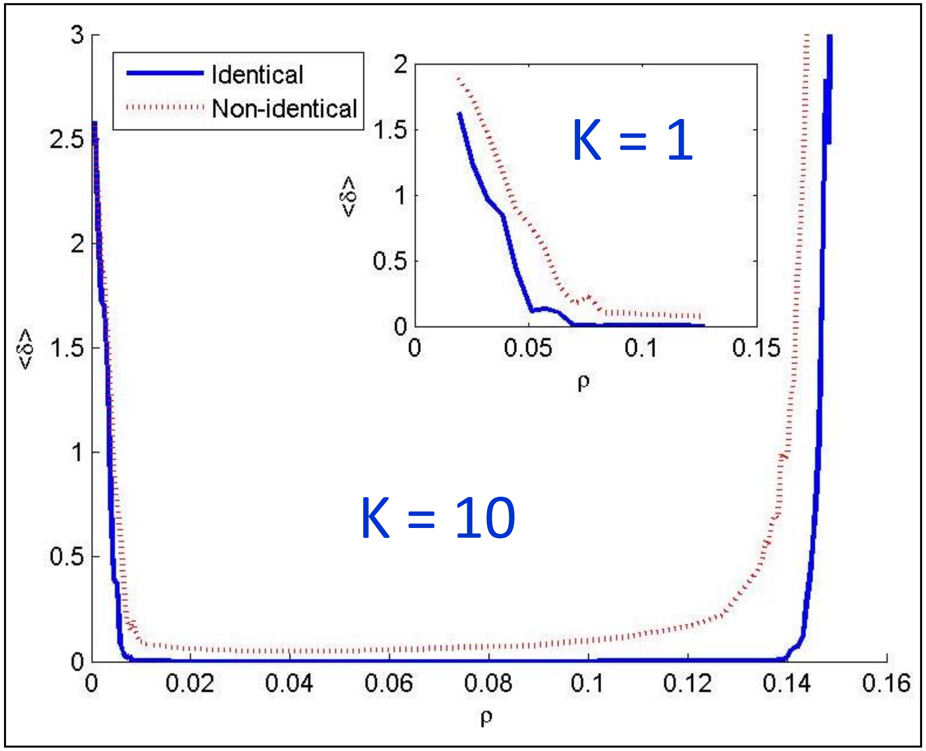
- K = 1

$$\rho_c = \frac{\alpha_1}{\pi r^2 K} \cong 0.07$$

- K = 10

$$\rho_{c_1} = \frac{\alpha_1}{\pi r^2 K} \cong 0.007$$

$$\rho_{c_2} = \frac{\alpha_2}{\pi r^2 K} \cong 0.1299$$

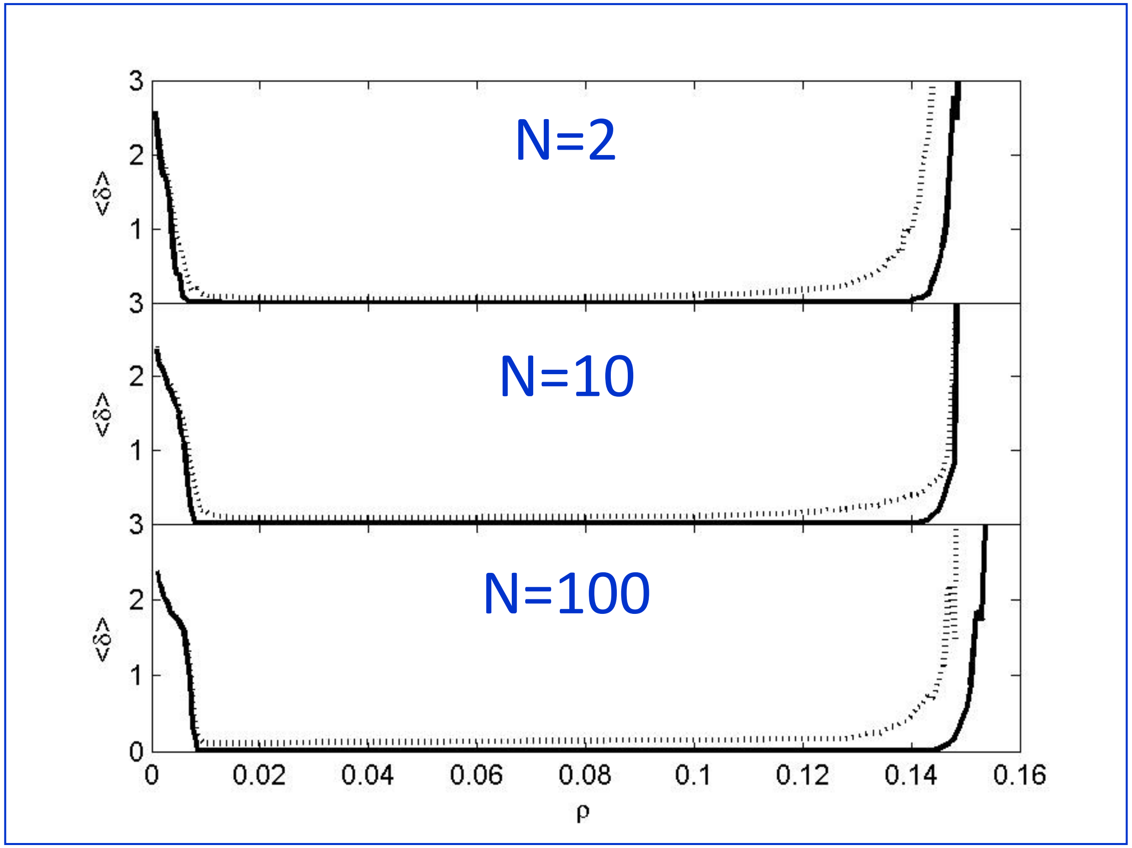


N=2, $\Delta t_M = \Delta t_s = 10^{-3}$, v=1,r=1

Synchronization conditions under FSH

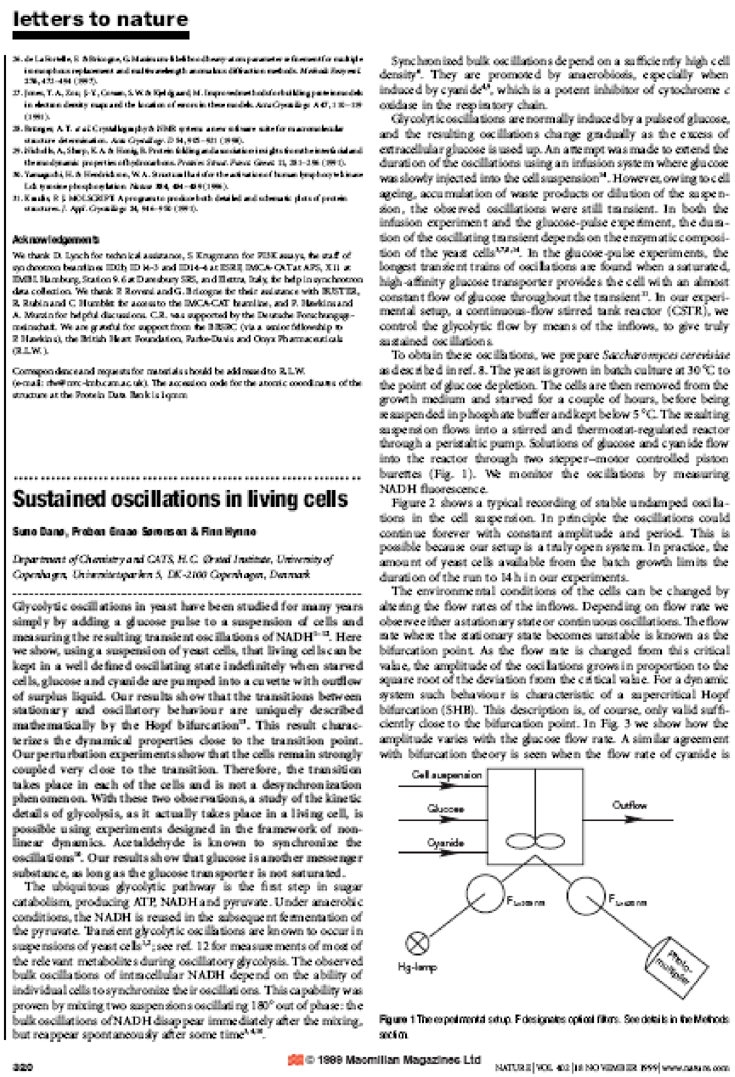
$$\frac{\alpha_1}{\pi r^2 K} = \rho_{c_1} < \rho < \rho_{c_2} = \frac{\alpha_2}{\pi r^2 K}$$

→ critical values do not depend on N



$\Delta t_M = \Delta t_s = 10^{-3}$, v=1,r=1, K=10

Sustained oscillations in living cells [Dano et al. *Nature*, 1999]



letters to nature

Sustained oscillations in living cells

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Glycolytic oscillations in yeast have been studied for many years simply by adding a glucose pulse to a suspension of cells and measuring the resulting transient oscillations of NADH. Here we show, using a suspension of yeast cells, that living cells can be kept in a well defined oscillating state indefinitely when starved cells, glucose and cyanide are pumped into a cuvette with outflow of surplus liquid. Our results show that the transitions between stationary and oscillatory behaviour are uniquely described mathematically by the Hopf bifurcation. This result characterizes the dynamical properties close to the transition point. Our perturbation experiments show that the cells remain strongly coupled very close to the transition. Therefore, the transition takes place in each of the cells and is not a desynchronization phenomenon. With these two observations, a study of the kinetic details of glycolysis, as it actually takes place in a living cell, is possible using experiments designed in the framework of non-linear dynamics. Acetaldehyde is known to synchronize the oscillations. Our results show that glucose is another messenger substance, as long as the glucose transporter is not saturated.

The ubiquitous glycolytic pathway is the first step in sugar catabolism, producing ATP, NADH and pyruvate. Under anaerobic conditions, the NADH is reused in the subsequent fermentation of the pyruvate. Transient glycolytic oscillations are known to occur in suspensions of yeast cells; see ref. 12 for measurements of most of the relevant metabolites during oscillatory glycolysis. The observed bulk oscillations of intracellular NADH depend on the ability of individual cells to synchronize their oscillations. This capability was proven by mixing two suspensions oscillating 180° out of phase: the bulk oscillations of NADH disappear immediately after the mixing, but reappear spontaneously after some time.

Synchronized bulk oscillations depend on a sufficiently high cell density. They are promoted by anaerobiosis, especially when induced by cyanide, which is a potent inhibitor of cytochrome *c* oxidase in the respiratory chain.

Glycolytic oscillations are normally induced by a pulse of glucose, and the resulting oscillations change gradually as the excess of extracellular glucose is used up. An attempt was made to extend the duration of the oscillations using an infusion system where glucose was slowly injected into the cell suspension. However, owing to cell ageing, accumulation of waste products or dilution of the suspension, the observed oscillations were still transient. In both the infusion experiment and the glucose-pulse experiment, the duration of the oscillating transient depends on the enzymatic composition of the yeast cells. In the glucose-pulse experiments, the longest transient trains of oscillations are found when a saturated, high-affinity glucose transporter provides the cell with an almost constant flow of glucose throughout the transient. In our experimental setup, a continuous-flow stirred tank reactor (CSTR), we control the glycolytic flow by means of the inflows, to give truly sustained oscillations.

To obtain these oscillations, we prepare *Saccharomyces cerevisiae* as described in ref. 8. The yeast is grown in batch culture at 30 °C to the point of glucose depletion. The cells are then removed from the growth medium and starved for a couple of hours, before being resuspended in phosphate buffer and kept below 5 °C. The resulting suspension flows into a stirred and thermostat-regulated reactor through a peristaltic pump. Solutions of glucose and cyanide flow into the reactor through two stepper–motor controlled piston burettes (Fig. 1). We monitor the oscillations by measuring NADH fluorescence.

Figure 2 shows a typical recording of stable undamped oscillations in the cell suspension. In principle the oscillations could continue forever with constant amplitude and period. This is possible because our setup is a truly open system. In practice, the amount of yeast cells available from the batch growth limits the duration of the run to 14 h in our experiments.

The environmental conditions of the cells can be changed by altering the flow rates of the inflows. Depending on flow rate we observe either a stationary state or continuous oscillations. The flow rate where the stationary state becomes unstable is known as the bifurcation point. As the flow rate is changed from this critical value, the amplitude of the oscillations grows in proportion to the square root of the deviation from the critical value. For a dynamic system such behaviour is characteristic of a supercritical Hopf bifurcation (SHB). This description is, of course, only valid sufficiently close to the bifurcation point. In Fig. 3 we show how the amplitude varies with the glucose flow rate. A similar agreement with bifurcation theory is seen when the flow rate of cyanide is

Figure 1 The experimental setup. F designates optical filters. See details in the Methods section.

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"Synchronized bulk oscillations depend on a sufficiently high cell density"

Contents

- Models of temporal networks
- Case studies of synchronization in temporal networks
  - blinking networks
  - activity driven networks
  - networks of mobile agents
  - adaptive networks

Adaptive networks: weight evolution

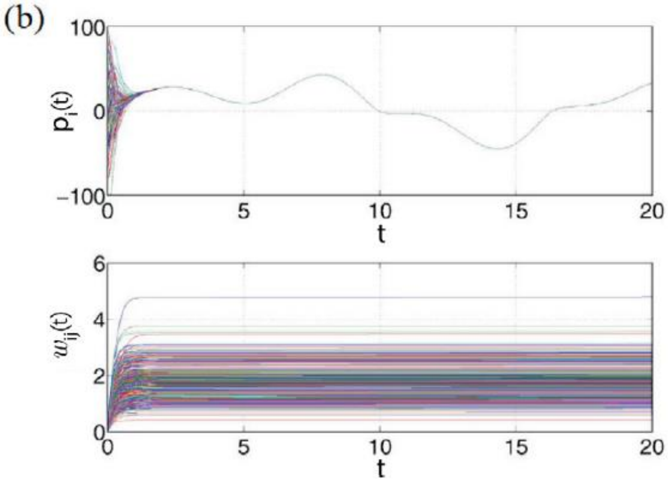
- Dynamics

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + \sum_{i=1}^{N} w_{ij}(t)(h(\mathbf{x}_j) - h(\mathbf{x}_i)),$$

- Dynamics of the weights

$$\dot{w}_{ij} = \mu \left\| h(\mathbf{x}_j) - h(\mathbf{x}_i) \right\|$$

- Global asymptotic stability via Lyapunov techniques if the function f is QUAD + (mild) conditions on the network



Adaptive networks: edge snapping

- Dynamics

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + \epsilon \sum_{i=1}^{N} w_{ij}(t)(h(\mathbf{x}_j) - h(\mathbf{x}_i)),$$

- Dynamics of the weight

$$\ddot{w}_{ij} + \gamma \dot{w}_{ij} + \frac{\partial}{\partial w_{ij}} V(w_{ij}) = g(\mathbf{e}_{ij}),$$

- Error function

$$g(\mathbf{e}_{ij}) = \|\mathbf{x}_j - \mathbf{x}_i\|^2$$

- Potential function

$$V(w_{ij}) = b w_{ij}^2 (w_{ij} - 1)^2,$$

Summary

- Only a few examples of temporal networks
- Variety of behaviors, due to the interplay between the time scales at work in the system
- Adaptivity: engineering the system to achieve synchronization